

## Polynômes minimaux

Calculer et factoriser le polynôme minimal de chacune des matrices suivantes (sachant que les racines de ces polynômes sont des entiers relatifs).

$$A_0 = \begin{pmatrix} 0 & 0 & 0 \\ 4 & -8 & 6 \\ 6 & -12 & 9 \end{pmatrix}$$

$$A_8 = \begin{pmatrix} 0 & -2 & 2 \\ -7 & -11 & 10 \\ -8 & -14 & 13 \end{pmatrix}$$

$$A_{16} = \begin{pmatrix} -3 & 12 & -8 \\ -3 & 9 & -4 \\ 0 & 0 & 3 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 3 & 4 & -4 \\ -4 & -5 & 4 \\ -2 & -2 & 1 \end{pmatrix}$$

$$A_9 = \begin{pmatrix} -7 & -8 & 8 \\ 2 & -1 & 0 \\ -2 & -6 & 5 \end{pmatrix}$$

$$A_{17} = \begin{pmatrix} -6 & 12 & -8 \\ 1 & -2 & 2 \\ 6 & -12 & 9 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -2 & -4 & 4 \\ 4 & 6 & -4 \\ 2 & 2 & 0 \end{pmatrix}$$

$$A_{10} = \begin{pmatrix} 8 & 10 & -10 \\ -1 & 3 & -2 \\ 4 & 10 & -9 \end{pmatrix}$$

$$A_{18} = \begin{pmatrix} 0 & 1 & 0 \\ 7 & 10 & -8 \\ 8 & 12 & -9 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} -4 & -6 & 6 \\ 6 & 8 & -6 \\ 3 & 3 & -1 \end{pmatrix}$$

$$A_{11} = \begin{pmatrix} 8 & 8 & -8 \\ -5 & -3 & 4 \\ -1 & 1 & 0 \end{pmatrix}$$

$$A_{19} = \begin{pmatrix} 0 & 1 & 0 \\ 7 & 10 & -8 \\ 8 & 12 & -9 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} 2 & 0 & 0 \\ -4 & 10 & -6 \\ -6 & 12 & -7 \end{pmatrix}$$

$$A_{12} = \begin{pmatrix} 5 & 4 & -4 \\ -7 & -8 & 8 \\ -5 & -7 & 7 \end{pmatrix}$$

$$A_{20} = \begin{pmatrix} 4 & 9 & -8 \\ -1 & -2 & 0 \\ 4 & 8 & -9 \end{pmatrix}$$

$$A_5 = \begin{pmatrix} -2 & 0 & 0 \\ -4 & 6 & -6 \\ -6 & 12 & -11 \end{pmatrix}$$

$$A_{13} = \begin{pmatrix} -6 & -8 & 8 \\ 2 & 0 & 0 \\ -2 & -6 & 6 \end{pmatrix}$$

$$A_{21} = \begin{pmatrix} -2 & -3 & 4 \\ 11 & 16 & -12 \\ 10 & 14 & -9 \end{pmatrix}$$

$$A_6 = \begin{pmatrix} 2 & 4 & -4 \\ -4 & -6 & 4 \\ -2 & -2 & 0 \end{pmatrix}$$

$$A_{14} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$A_{22} = \begin{pmatrix} -5 & -3 & 4 \\ 11 & 13 & -12 \\ 10 & 14 & -12 \end{pmatrix}$$

$$A_7 = \begin{pmatrix} 5 & 4 & -4 \\ -4 & -3 & 4 \\ -2 & -2 & 3 \end{pmatrix}$$

$$A_{15} = \begin{pmatrix} 1 & 2 & -2 \\ -8 & -13 & 10 \\ -7 & -11 & 8 \end{pmatrix}$$

$$A_{23} = \begin{pmatrix} -5 & -3 & 4 \\ 11 & 13 & -12 \\ 10 & 14 & -12 \end{pmatrix}$$

Solutions :

$$\mu_0 = x^2 - x = (x - 1) \cdot x$$

$$\mu_1 = x^2 - 1 = (x - 1) \cdot (x + 1)$$

$$\mu_2 = x^2 - 2x = (x - 2) \cdot x$$

$$\mu_3 = x^2 - x - 2 = (x - 2) \cdot (x + 1)$$

$$\mu_4 = x^2 - 3x + 2 = (x - 2) \cdot (x - 1)$$

$$\mu_5 = x^2 + 5x + 6 = (x + 2) \cdot (x + 3)$$

$$\mu_6 = x^2 + 2x = x \cdot (x + 2)$$

$$\mu_7 = x^2 - 4x + 3 = (x - 3) \cdot (x - 1)$$

$$\mu_8 = x^3 - 2x^2 - x + 2 = (x - 2) \cdot (x - 1) \cdot (x + 1)$$

$$\mu_9 = x^3 + 3x^2 - x - 3 = (x - 1) \cdot (x + 1) \cdot (x + 3)$$

$$\mu_{10} = x^3 - 2x^2 - 5x + 6 = (x - 3) \cdot (x - 1) \cdot (x + 2)$$

$$\mu_{11} = x^3 - 5x^2 + 4x = (x - 4) \cdot (x - 1) \cdot x$$

$$\mu_{12} = x^3 - 4x^2 + 3x = (x - 3) \cdot (x - 1) \cdot x$$

$$\mu_{13} = x^3 - 4x = (x - 2) \cdot x \cdot (x + 2)$$

$$\mu_{14} = x^3 - 2x^2 - 9x + 18 = (x - 3) \cdot (x - 2) \cdot (x + 3)$$

$$\mu_{15} = x^3 + 4x^2 + 3x = x \cdot (x + 1) \cdot (x + 3)$$

$$\mu_{16} = x^2 - 6x + 9 = (x - 3)^2$$

$$\mu_{17} = x^3 - x^2 = (x - 1) \cdot x^2$$

$$\mu_{18} = x^3 - x^2 - x + 1 = (x + 1) \cdot (x - 1)^2$$

$$\mu_{19} = x^3 - x^2 - x + 1 = (x + 1) \cdot (x - 1)^2$$

$$\mu_{20} = x^3 + 7x^2 + 15x + 9 = (x + 1) \cdot (x + 3)^2$$

$$\mu_{21} = x^3 - 5x^2 + 3x + 9 = (x + 1) \cdot (x - 3)^2$$

$$\mu_{22} = x^3 + 4x^2 = (x + 4) \cdot x^2$$

$$\mu_{23} = x^3 + 4x^2 = (x + 4) \cdot x^2$$